Concurrent Implementation of the Complementary Operators Method in 2-D Space

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***Abstract*— This letter presents a novel implementation of the complementary operators method (COM). In contrast to the original implementation of COM, in this approach, the complementary operators are applied concurrently on a set of fields in the boundary region. This not only allows the cancellation of first-order reflections, but the reduction of all subsequent reflections as well. A numerical example is given to validate this new scheme and to show that it results in unprecedented suppression of artificial reflections.**

***Index Terms*—Finite-difference time domain, numerical methods, wave propagation.**

I. Introduction

The complementary operators method (COM) was originally introduced as a mesh truncation technique for open-region finite-difference time-domain (FDTD) simulations [1], [2]. The basic premise of COM is the cancellation of the first-order reflection that arise from the truncation of the computational domain. This cancellation is made possible by averaging two independent solutions of the problem. These two solutions are obtained by imposing boundary operators that are complementary to each other, in the sense that the errors generated by the two operators are equal in magnitude but 180° out of phase. As a result of the averaging process, the first-order reflections, consisting of either evanescent or traveling waves, are annihilated.

Although COM requires two independent solutions of the problem, which lead to doubling the total operation count, it was still found to be highly effective and efficient when compared to other available mesh truncation techniques [3]. Ideally, however, one would like to perform the averaging of the two complementary solutions at the boundary, and in a single simulation. If this was possible, then not only will the operation count remain unchanged, but also first- and second-order reflections from the boundary and corner regions will be canceled. This, consequently, leads to the reduction of all subsequent secondary reflections (the reflections of the first reflections). In this letter, we show that such an operation is possible, and that it leads to further suppression of artificial reflections in comparison to that obtained using the original implementation of COM.

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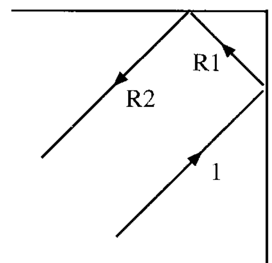

Fig. 1. Corner region reflections.

II. Complementary Operators Method

When basing COM on Higdon's Nth-order operators [4], and assuming a terminal boundary perpendicular to the x-axis, we have the following two complementary operators:

$$B_N^- = \partial_x \prod_{i=1}^{N} \left(\partial_x + \frac{1}{c}\,\partial_t + \alpha_i\right) U = 0 \tag{1}$$

$$B_N^+ = \partial_t \prod_{i=1}^{N} \left(\partial_x + \frac{1}{c}\,\partial_t + \alpha_i\right) U = 0 \tag{2}$$

where U is the unknown field, c is the speed of light, and the parameter α_i ensures a less than unity reflection coefficient for evanescent waves. We denote the corresponding reflection coefficients, respectively, by $-R$ and R. Discretization of these operators is straightforward and can be found in [4].

The averaging of the two solutions obtained from applying each of the two operators separately gives a solution containing only second-order reflections, including those that arise from corner regions, which we schematically show in Fig. 1.

The corner reflections constitute the second most dominant reflections because they reach the observation point faster than multiple reflections due to the scatterer [1]. To cancel these reflections, four solutions instead of two need to be averaged, with each requiring an independent simulation. For each simulation, one needs to impose a unique combination of B_N^- and B_N^+ over the four sides of the outer boundary. These combinations are shown in Fig. 2 for each of the four simulations, where $-$ corresponds to B_N^- and $+$ corresponds to B_N^+.

For further illustration, we show in Table I the magnitudes of the first- and second-order reflections due to the upper-right corner (assuming an incident pulse of unity magnitude), for each of the four needed solutions. Notice that the average of all the values in the third column eliminates the corner reflections.

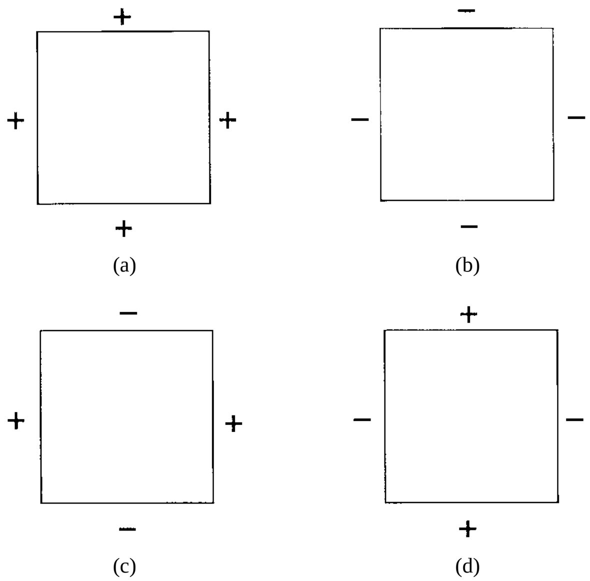

Fig. 2. Illustration showing the four different combinations of boundary operators needed to annihilate corner reflections.

TABLE I
MAGNITUDES OF CORNER REGION REFLECTIONS

	1st reflection (R1)	2nd reflection (R2)
(a)	R	R^2
(b)	$-R$	R^2
(c)	R	$-R^2$
(d)	$-R$	$-R^2$

Unfortunately, this procedure results in quadrupling the operation count, while at the same time not allowing for any further cancellation of any subsequent reflections.

III. Concurrent Complementary Operators Method

The basic objective of the concurrent implementation of COM is to cancel the first-order reflections from the side boundaries and the second-order reflections from the corner regions, in a *single* computer run.

To this end, we first divide the geometry into two regions: a boundary zone and an interior region, as shown in Fig. 3. The interior region includes the scattering object and any localized sources. In the boundary zone, instead of defining one storage location for each of E_z, H_x, and H_y (as in TM polarization case, for example), we allocate four storage locations to each field. Then, we perform a single simulation of the problem where each of the four locations corresponding to a single field is updated using one of the four boundary combinations shown in Fig. 2. Within the interior region, only a single value for each of the field components is stored as in typical FDTD implementation. What we have done thus far can be thought of as carrying out four different simulations in the boundary zone.

The next step is to link the two regions. This is performed by averaging the four values obtained for each field at an interface perimeter that is placed immediately to the inside of the boundary zone. This averaging is performed at each

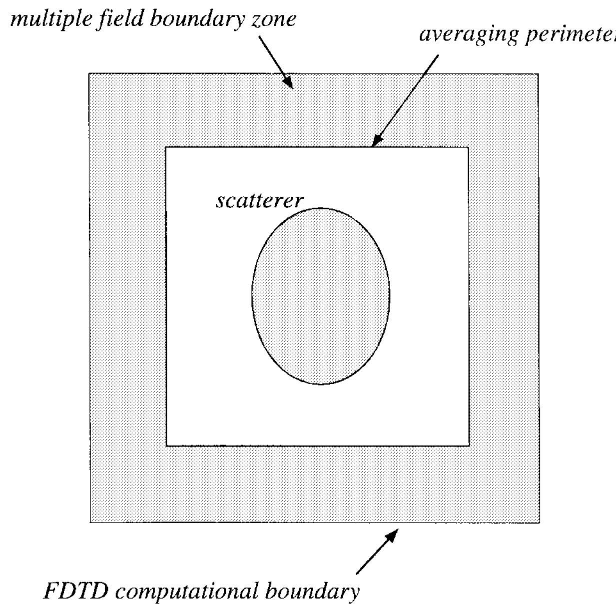

Fig. 3. Decomposition of the FDTD computational space into a boundary zone and an interior region containing the scatterer and localized sources.

time step. The width of the boundary zone is strictly defined by the length of the stencil needed to discretize the boundary operators in (1) or (2). Notice that if the averaging is carried out at a perimeter placed within the boundary zone, then the solution becomes unstable because the discretized Maxwell's equations experience an additional artificial source.

IV. Numerical Validation

To demonstrate the validity of the concurrent implementation of complementary operators, we consider a current line source in two-dimensional (2-D) space (TM polarization), radiating in free space. We consider a computational domain of size 31 × 31 where the source is centered at (16, 16) and a uniform space step of size 0.015 m. The temporal form of the source is a compact pulse given by $f(t) = h(t) * h(t)$, where $*$ denotes the convolution operation, and $h(t)$ is defined over $0 \le t \le t_o$ and is given by

$$h(t) = 15\omega_1 \sin(\omega_1 t) - \omega_2 \sin(\omega_2 t) + \omega_3 \sin(\omega_3 t) \quad (3)$$

where $t_o = 10^{-9}$ and $\omega_i = 2\pi i/t_o$, $i = 1, 2, 3$.

An observation point is chosen at (10, 10). Fig. 4 shows the normalized error in E_z defined according to

$$\text{Error}(t) = \frac{|u(t) - u^{ref}(t)|}{\max[|u^{ref}(t)|]} \quad (4)$$

where $u(t)$ is the time signature of the output pulse using this method, which we refer to as C-COM, or the other techniques used for comparison. u^{ref} is the reference solution obtained in a domain large enough such that the reflections from the terminal boundaries are not present in the solution. Higdon's fourth-order operators are used for COM4 and C-COM4 [$N = 3$ in (1) and (2)]. In Fig. 4, two results are given for C-COM4 simulations, labeled as C-COM4 (7) and

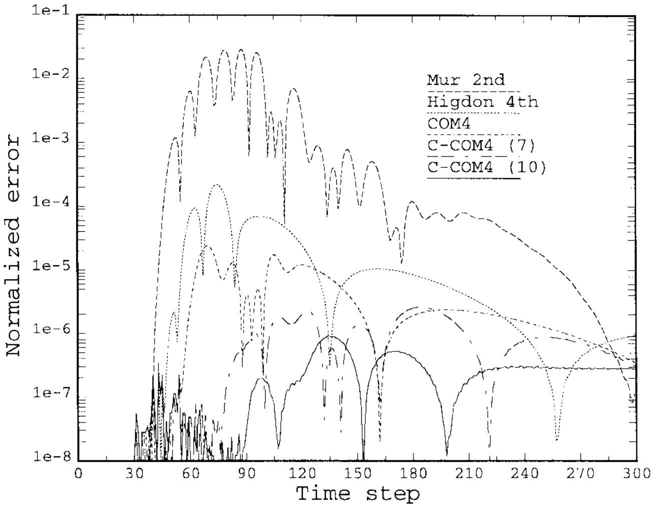

Fig. 4. Normalized error in E_z obtained using Mur second, Higdon fourth, COM4, and C-COM4.

C-COM4 (10), corresponding to a boundary zone of 7- and 10-cells wide, respectively.

These numerical results show that for the numerical problem considered, an appreciable improvement was achieved in comparison to that obtained using the original implementation of COM. The major advantage of this technique, however, is that it is able to achieve such accuracy without resorting to two independent simulations.

V. Conclusion

A novel implementation of COM is presented based on the concurrent application of complementary operators. The method is very simple to implement since it is based on one-way wave equations such as Higdon's boundary operators. A numerical example is illustrated showing effective suppression of artificial reflections in a single computer run.

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